

Answer all of the questions. Calculators, pagers and mobile telephones are NOT allowed.

1. (4 pts) Find the following limits.

$$(a) \lim_{x \rightarrow \infty} (1 + e^{-x})^{\exp(x)}$$

$$(b) \lim_{x \rightarrow 1} \left( \frac{1}{1-x} - \frac{1}{\ln x} \right)$$

2. (12 pts) Evaluate the following integrals.

$$(a) \int \frac{x^3 + 1}{(x^2 + 1)^2} dx$$

$$(b) \int e^x \sin 2x dx$$

$$(c) \int \sin^5 x \cos^2 x dx$$

$$(d) \int \sqrt{1 + \sqrt{x}} dx$$

3. (4 pts) Determine whether the improper integral converges or diverges; if it converges, find its value.

$$\int_1^2 \frac{dx}{\sqrt{-x^2 + 4x - 3}}$$

4. (5 pts) Let  $C$  be the curve given by the parametric equations

$$x = 1 + \sin^{-1} t \quad y = 1 - \cos^{-1} t, \quad 0 \leq t \leq 1.$$

(a) Find the length of  $C$ .

(b) Find the equation of the tangent line at the point corresponding to  $t = 0$ .

(c) Find  $\frac{d^2y}{dx^2}$